Resonant tunneling in a pulsed phonon field

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We theoretically investigate resonant tunneling through a single level assisted by short LO phonon pulses. The analysis is based on the recently developed nonequilibrium linked-cluster expansion [P. Král, Phys. Rev. B **56**, 7293 (1997)], extended in this work to transient situations. The nonequilibrium spectral function for the resonance displays the formation and decay of the phonon sidebands on ultrashort time scales. The time-dependent tunneling current through the individual phonon satellites reflects this quasiparticle formation by oscillations, whose time scale is set by the frequency of the phonon field and its harmonics. These oscillations are washed out at elevated temperatures. [S0163-1829(99)04208-3]

I. INTRODUCTION

Carrier dynamics at very short time scales displays many intriguing phenomena. Basic concepts, such as energy conservation in a scattering process, must be carefully rescrutinized. Several recent optical experiments on femtosecond time scale have illustrated these effects.[1,2] Another important example is the work of Fürst *et al.*,[3] where the phonon-emission related replica of the initial electron distribution, centered at E_0, settle to the energy $E_0 - n\hbar\omega_{LO}$ only after several phonon oscillation periods. A good account of this experiment can be given with the *exact* one-dimensional (1D) theory for the time-dependent electronic distribution function in the ultrashort time scale, due to Meden *et al.*[4] This is the time scale for quasiparticle formation and the standard Boltzmann picture, which assumes well-defined quasiparticles cannot be applied. Thus a proper theoretical description must account for non-Markovian effects, such as retardation and/or memory effects, in the collision term. Examples of such theories are those based on the density-matrix[5] or nonequilibrium Green functions,[6] which have successfully explained some of the above-mentioned experimental features.[1,2] These theories, however, often result in very complicated expressions, and a numerical evaluation requires many approximations.

The purpose of this work is to introduce a different theoretical approach, which allows a relatively straightforward numerical evaluation. We apply the method to a mesoscopic transport situation, which represents a generalization of recent studies of resonant tunneling assisted by quasiadiabatic pulses of hot LA phonons.[7,8] In addition, as shall be seen below, the physics bears a similarity to the optical measurements of Ref. 3. Specifically, we consider a dc-biased resonant tunneling system, which is excited by short pulses of nonequilibrium LO phonons. Experimentally, this might be realized by subjecting the sample to suitable light pulses, the propagation of which is known to be accompanied by lattice vibrations.[9]

It is well known, both experimentally[10] and theoretically,[11] that optical phonons lead to additional structure in the measured IV curve of resonant tunneling systems, i.e., secondary maxima at voltages determined by the LO-phonon frequency. To investigate the time-dependent formation of these structures caused by *pulsed* phonon fields, we generalize the nonequilibrium linked-cluster expansion (NLCE) of Ref. 12 (to be referred to as I from this on) to the time domain (NCLET). This method is applied to study the fast electron dynamics in the resonant tunneling system, induced by the nonequilibrium phonon field with a zero coherent part, but *fast time fluctuations*. This distinguishes our work from the existing literature on time-dependent behavior of mesoscopic systems[13] (up-to-date reviews are available, e.g., in Ref. 14). We calculate the time-dependent formation of satellite peaks in the spectral function, induced by the phonon pulses, and evaluate the related transient resonant tunneling current through these individual peaks.

The paper is organized as follows. In Sec. II the NLCET method for the Green functions is developed. In Sec. III we apply this approach to a dc-biased resonant tunneling diode, which is exposed to a very short phonon pulse. Section IV is devoted to numerical results for the nonequilibrium spectra and currents.

II. NONEQUILIBRIUM LINKED CLUSTER EXPANSION IN TIMES

The NLCE method combines the nonequilibrium Green functions[6,15,16] (NGF) with the equilibrium linked cluster expansion (LCE).[17] The appealing feature of this connection is that all Feynman diagrams in NGF are topologically equivalent to their equilibrium counterparts, which, on the other hand, are also used in the LCE method (see, e.g., pp 524–555 of Ref. 17, whose notation we follow closely). Thus all results for the NLCE method are readily at our disposal, and, in particular, the nonequilibrium interacting electron correlation functions $G^<$ can be rather simply evaluated in terms of nonequilibrium noninteracting functions $G_0^<$. In the first-order linked cluster approximation, NLCE gives reliable results for moderate interaction strengths.[12] For electrons coupled to LO phonons, the dimensionless electron-phonon coupling constant g, defined below Eq. (4), should satisfy $g \approx 0.3-1$ (depending on the temperature). Here we generalize NLCE to the time domain (NLCET) and use it to evalu-

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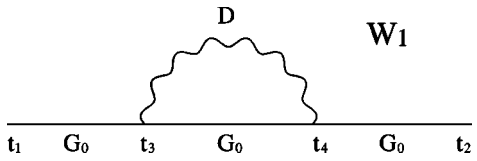


FIG. 1. The first-order linked cluster diagram W_1. An analytic continuation to real times gives the expression for $\Delta W^<(t_1,t_2)$ used in the text.

ate interacting electron correlation functions in a pulsed phonon field.

The present time-dependent situation does not lead to any structural changes in the theory from I; it is sufficient to consider the time coordinates (t_1,t_2) as independent instead of the t_1-t_2 dependence used in steady states (the transient phonon field breaks the time-translational invariance). The nonequilibrium electron correlation function $G^<(t_1,t_2) \equiv 1/\hbar\langle c^\dagger(t_1)c(t_2)\rangle$ in real times can be expanded in terms of the correlation parts of the cluster diagrams $W_n^{<,>}(t_1,t_2)$, formed by nth order terms in the interaction part of the Hamiltonian (W_1 for electron-phonon coupling is in Fig. 1). Explicit expressions for these correlation parts can be obtained by analytic continuation performed with the Langreth rules.[6,18] In full analog with the steady-state situation in I, the expansion for $G^<$ is then exponentially resummed in terms of the coefficients $F_n^<$ [$G^>(t_1,t_2)$ can be obtained similarly]:

$$G^<(t_1,t_2)=\sum_{n=0}^{\infty} W_n^<(t_1,t_2)$$

$$=G_0^<(t_1-t_2)\exp\left(\sum_{n=1}^{\infty} F_n^<(t_1,t_2)\right), \qquad (1)$$

given by

$$F_1^<(t_1,t_2)=\frac{W_1^<(t_1,t_2)}{G_0^<(t_1-t_2)},$$

$$F_2^<(t_1,t_2)=\frac{W_2^<(t_1,t_2)}{G_0^<(t_1-t_2)}-\tfrac{1}{2}F_1^<(t_1,t_2)^2, \ \ldots . \qquad (2)$$

The noninteracting correlation functions $G_0^{<,>}(t_1-t_2)$ are time homogeneous, since they correspond to the underlying nonequilibrium *steady* state, without the time-dependent phonon-field.

The correlation function in (1) can be transformed to center of mass (CMS) coordinates $\tau=t_1-t_2$, $T=(t_1+t_2/2)$:

$$G^<(\tau,T)=\sum_{n=0}^{\infty} W_n^<(\tau,T)=G_0^<(\tau)\exp\left(\sum_{n=1}^{\infty} F_n^<(\tau,T)\right),$$

$$\tau=t_1-t_2, \quad T=\frac{t_1+t_2}{2}. \qquad (3)$$

After a Fourier transform over the difference time τ, the correlation functions $G^{<,>}(\omega,T)$ provide the spectral and time information by ω and T, respectively. The expansion of the nonequilibrium spectral function $A(\omega,T)=G^>(\omega,T)+G^<(\omega,T)$ can be obtained in a similar fashion. It may assume negative values, just like the quasidistribution $f(\omega,T)=G^<(\omega,T)/[G^>(\omega,T)+G^<(\omega,T)]$, which is an analog of the Wigner function in transport. One should also note that a low-order truncation of the linked cluster expansion is not guaranteed to lead to a conserving approximation for the quasidistribution, and, in particular, under time-dependent situations careful checks are necessary. In the present paper, however, all calculated quantities derive from the nonequilibrium spectral function, which satisfies the sum rule $\int \frac{d\omega}{2\pi} A(\omega,T)=1$, and the conservation rules are satisfied by construction.

III. APPLICATION TO TUNNELING

We apply the NLCET to a resonant tunneling system coupled to LO phonons, which was examined in steady-state situations in I. In the present work the phonon population has a pulse form, but the coherent part of the phonon field is zero.[19]

A. Model

The model consists of a quantum well with one level coupled to two wide-band reservoirs and one LO phonon mode. The Hamiltonian is thus

$$H=\sum_{k;\,\alpha=L,R} E_{k,\alpha}c_{k,\alpha}^\dagger c_{k,\alpha}+E_0 d^\dagger d$$

$$+\sum_{k;\alpha=L,R}\gamma_{k;\alpha}(c_{k,\alpha}^\dagger d+\text{H.c.})+\hbar\omega_0 b^\dagger b+M d^\dagger d(b+b^\dagger). \qquad (4)$$

Here $E_{k,\alpha=L,R}$ are electron energies in the left (right) reservoirs, E_0 is the energy of the level, and the parameters $\gamma_{k,\alpha=L,R}$ give the coupling of the level to the reservoirs. The phonon energy $\hbar\omega_0$ and the electron-phonon matrix element M define the interaction strength $g=(M/\hbar\omega_0)^2$.

Under dc bias, the electrochemical potentials in the reservoirs $\mu_{L,R}$ shift in opposite directions by equal amounts. Then the free electron correlation functions and propagators for the electrons on the level are[20] ($\tau=t_1-t_2$)

$$G_0^<(\tau)=\int\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{\Gamma/2}{(\hbar\omega-E_0)^2+\Gamma^2/4}$$

$$\times[n_{FD}(\hbar\omega-\mu_L)+n_{FD}(\hbar\omega-\mu_R)],$$

$$G_0^>(\tau)=\int\frac{d\omega}{2\pi}e^{-i\omega\tau}\frac{\Gamma/2}{(\hbar\omega-E_0)^2+\Gamma^2/4}$$

$$\times[2-n_{FD}(\hbar\omega-\mu_L)-n_{FD}(\hbar\omega-\mu_R)],$$

$$G_0^{r,a}(\tau)=\mp i\,\theta(\pm\tau)[G_0^>(\tau)+G_0^<(\tau)]$$

$$=\mp\frac{i}{\hbar}\theta(\pm\tau)e^{-\frac{i}{\hbar}E_0\tau\mp\frac{\Gamma}{2\hbar}\tau}, \qquad (5)$$

where Γ characterizes the coupling to the reservoirs, assumed symmetric as in I. The correlation functions and propagators for the equilibrium phonons are

$$D^{<}(\tau)=\frac{i}{\hbar}\{e^{-i\omega_0\tau}n_{BE}(\omega_0)+e^{i\omega_0\tau}[1+n_{BE}(\omega_0)]\},$$

$$D^{>}(\tau)=\frac{i}{\hbar}\{e^{-i\omega_0\tau}[1+n_{BE}(\omega_0)]+e^{i\omega_0\tau}n_{BE}(\omega_0)\},$$

$$D^{r,a}(\tau)=\mp i\,\theta(\pm\tau)[D^{>}(\tau)-D^{<}(\tau)]$$
$$=\mp\frac{2}{\hbar}\theta(\pm\tau)\sin(\omega_0\tau). \tag{6}$$

As usual, negative frequencies, related to phonon emission, are eliminated by $n_{BE}(-\omega_0)=-[1+n_{BE}(\omega_0)]$. In the presence of the pulsed phonon field, all the correlation functions separately depend on both time variables (t_1,t_2). Below we *construct* the pulsed phonon correlation functions, by neglecting back action of the tunneling electrons on the phonons, and evaluate the electron correlation functions in this field by the NLCET method.

B. The time-dependent phonon field

As mentioned in the Introduction, subjecting a semiconductor sample to light pulse can result in a propagating phonon pulse. Viewed from the resonant-tunneling diode, the propagating phonon pulse implies that within a certain time-interval the phonon distribution, which interacts with the diode, deviates from its equilibrium value. Alternatively, a pulsed photon-field that couples directly with the tunneling electrons, leads to similar physics.[21] In order to test the NLCET method in this transient problem, we neglect, for simplicity, the coherent component of the excitation boson field, which can be described by simpler methods, and consider only its component with a random phase.

A mathematical description of this physical situation can be achieved as follows. The population of the LO phonon field at frequency ω_0 suddenly increases at the time $T=0$ by $n_P(\omega_0)$ phonons and remains constant until $T=T_0$, when $n_P(\omega_0)$ is switched off. This process can be described by the nonequilibrium phonon distribution

$$f_P(\omega_0,T)=n_{BE}(\omega_0)+\Delta_{\omega_0}(T),$$

$$\Delta_{\omega_0}(T)=n_P(\omega_0)\;\;[\theta(T-T_0)-\theta(T)], \tag{7}$$

which can be used to construct the total phonon correlation functions $D^{<},D^{>}$. This construction, however, must be done carefully. In Kadanoff-Baym equations, two closely related approaches are used for constructing nonequilibrium correlation functions from the nonequilibrium distributions; the KB Ansatz[15] and the GKB Ansatz.[22] The NLCET method is not iterative and the Ansatz must be done on the nonequilibrium phonon field, which is not the product of the solution but enters *externally*. Therefore it seems to be worth to explore both approaches in this situation in Appendix. We realize that in NLCET the consistent approach is also the GKB Ansatz, giving the nonequilibrium phonon correlation functions in (A4)

$$\Delta D^{<}(\tau,T)=\Delta D^{>}(\tau,T)\approx\frac{2}{\hbar}\cos(\omega_0\tau)$$
$$\times\left[\theta(\tau)\Delta_{\omega_0}\left(T-\frac{\tau}{2}\right)+\theta(-\tau)\Delta_{\omega_0}\left(T+\frac{\tau}{2}\right)\right]. \tag{8}$$

Below, we prove that the structure of arguments in $\Delta D^{<,>}(\tau,T)$ assures causal phonon-induced observables.

C. Time-dependent $G^{<},G^{>}$

The phonon correlation functions in (8) can be used directly in the NLCET method to obtain the transient electron correlators $G^{<},G^{>}$. We consider only the lowest order term,

$$G^{<}(\tau,T)\approx G_0^{<}(\tau)\exp[F_1^{<}(\tau)+\Delta F_1^{<}(\tau,T)], \tag{9}$$

which can be factorized into a steady-state contribution

$$G^{<}_{\text{steady}}(\tau)=G_0^{<}(\tau)\exp[F_1^{<}(\tau)], \tag{10}$$

describing static tunneling modified by equilibrium phonons, and a time-dependent term, $\exp[\Delta F_1^{<}(\tau,T)]$. In the present study, which focuses on the time-dependent changes in the tunneling current, it is sufficient to approximate $G^{<}_{\text{steady}}\approx G_0^{<}$, i.e., the effects due to equilibrium phonons are not considered.[23]

The first order linked cluster factor W_1 in (2) is represented by the diagram in Fig. 1. The change of the related correlation part $\Delta W_1^{<}(t_1,t_2)$, induced by the phonon pulse, is obtained with the Langreth rules[6,18] [the expression for $\Delta W_1^{>}(t_1,t_2)$ is analogous]:

$$\Delta W_1^{<}(t_1,t_2)=M^2\int dt_3\int dt_4\{G_0^{<}(t_1-t_3)[G_0(t_3-t_4)\Delta D(t_3,t_4)]^a G_0^a(t_4-t_2)$$
$$+G_0^r(t_1-t_3)G_0^{<}(t_3-t_4)\Delta D^{<}(t_3,t_4)G_0^a(t_4-t_2)+G_0^r(t_1-t_3)[G_0(t_3-t_4)\Delta D(t_3,t_4)]^r G_0^{<}(t_4-t_2)\}. \tag{11}$$

Here the phonon Green functions ΔD are given by (8), and the free electron Green functions G_0 are given by the noninteracting solution (5), respectively. The integrals in (11) can be easily performed numerically, if the theta functions in (8) are taken as integration limits in (11), so that the phonon functions depend only on the time-translationally invariant cosines. From the change $\Delta W_1^<(t_1,t_2)$ the first order coefficient $\Delta F_1^<(\tau,T)$ in (3) results as [$\Delta F_1^>(\tau,T)$ and $\Delta A_1(\tau,T)$ can be obtained similarly]:

$$\Delta F_1^<(\tau,T)=\frac{\Delta W_1^<(\tau,T)}{G_0^<(\tau)}. \tag{12}$$

Substitution of these coefficients in (9) gives the first-order linked cluster approximation for the time-dependent correlation functions $G^<(\tau,T)$, $G^>(\tau,T)$. Physical observables calculated from these functions must be causal. In the next section we explicitly show that this is the case for the induced change in the current, if the phonon correlation functions (8) are used.

D. Time-dependent tunneling current

The expression for the time-dependent tunneling current can be obtained by the steps used in Sec. IV of Ref. 13. In the wide-band limit with symmetric coupling, the currents from the left and right reservoirs to the level satisfy $J_L(t)=-J_R(t)$. Therefore, it is very convenient to rearrange the current in a symmetrized way with respect to these reservoirs,[13,8] because in this form the terms explicitly involving the level population $G^<$ cancel. Then the current assumes a very simple form

$$\begin{aligned} J(t)&=\frac{J_L(t)-J_R(t)}{2}=\frac{e\Gamma}{4i}\int_{-\infty}^{\infty}d\bar{t}\,[\,\delta n_{FD}(t-\bar{t})G^a(\bar{t},t)+G^r(t,\bar{t})\,\delta n_{FD}(\bar{t}-t)\,] \\ &=\frac{e\Gamma}{4i}\int_{-\infty}^{\infty}d\tau[G^a(\tau,T=t+\tau/2)-G^r(\tau,T=t-\tau/2)]\,\delta n_{FD}(-\tau), \end{aligned} \tag{13}$$

where $\delta n_{FD}(t)=\int d\omega/2\pi e^{-i\omega t}[n_{FD}(\hbar\omega-\mu_L)-n_{FD}(\hbar\omega-\mu_R)]$ is a Fourier transform of the difference of two Fermi-Dirac distributions. The effect of this term is to average out the transient time oscillations from the propagators $G^{r,a}(\tau,T=t\pm\tau/2)$ and consequently from the current $J(t)$. The averaging is stronger the broader $\delta n_{FD}(\omega)$ is in energy, corresponding to higher biases or temperatures.

We next explicitly demonstrate the causality of the current $J(t)$ in (13): the current must satisfy $J(t<0)=J_0$, where J_0 is the static current before the phonon pulse. We observe first that the term in square brackets in the second line of (13) can be expressed in terms of the nonequilibrium spectrallike function:

$$\begin{aligned} \mathcal{A}(\tau,t)&\equiv i[G^r(\tau,T=t-\tau/2)-G^a(\tau,T=t+\tau/2)] \\ &=\theta(\tau)A(\tau,t-\tau/2)+\theta(-\tau)A(\tau,t+\tau/2). \end{aligned}$$

Note also that the time arguments have a very similar form to the GKB Ansatz for the phonon Green functions in (8). If $\mathcal{A}(\tau,t<0)=A_0(\tau)$, the proof is complete. The phonon GKB Ansatz makes $\Delta W_1^{<,>}(t_1,t_2)$ causal in the times t_1,t_2 [i.e., it vanishes if $t_1<0$ or $t_2<0$; this can be easily checked from Eq. (11)], and hence the spectral function $A(t_1,t_2)$ is causal in t_1,t_2. We examine now the first term, $\theta(\tau)A(\tau,t-\tau/2)$ (the second term gives the same result). By construction, it can be nonzero only if $\tau>0$. Then, for observation times $t<0$ (i.e., before the nonequilibrium phonon pulse is operative) it holds that $t-\tau/2\equiv t_2<0$ and consequently the spectral function has its equilibrium form, $A(\tau,t-\tau/2<0)=A_0(t_1-t_2)$. Therefore the current is equal the steady-state value $J(t)=J_0$, which explicitly proves its causality. On the other hand, the use of the KB Ansatz for the phonon functions (A1) would break this causality of $J(t)$, similarly as neglecting the shifts $\pm\tau/2$ in the time arguments of the propagators $G^{r,a}(\tau,t\pm\tau/2)$. The last approximation (see also Ref. 24) is based analogously as the KB Ansatz and it is equal the lowest order approximation in gradient expansions in the shifts $\pm\tau/2$, applied for slow external fields.[16,25,26] We caution against this expansion under transient conditions, since it is inevitably accompanied by noncausality.

The current can be given a more intuitive form with the help of the above explicitly t-causal spectral function $\mathcal{A}(\tau,t)$. It can be expressed in terms of the frequency representation of $\mathcal{A}(\omega,t)$ as follows:

$$J(t)=\frac{e\Gamma}{4\hbar}\int\frac{d\hbar\omega}{2\pi}\mathcal{A}(\omega,t)[n_{FD}(\hbar\omega-\mu_L)-n_{FD}(\hbar\omega-\mu_R)]. \tag{14}$$

It is important to distinguish the physical difference of the two functions $\mathcal{A}$ and A; the function A can be used to calculate (generalized) densities of states, but in the calculation of transient current one must use $\mathcal{A}$. Substitution of $\mathcal{A}(\omega,t)$ by $A(\omega,t)$ in the current formula (14) gives the above lowest order term in the gradient expansion.

IV. NUMERICAL RESULTS

We first investigate numerical results for the nonequilibrium spectral function $A(\omega,T)$. Later the transient tunneling current $J(t)$ is evaluated as well.

A. Nonequilibrium spectrum

In Fig. 2 the time evolution is shown for the function $A(\omega,T)$, calculated from expression (9) for a pulsed phonon field of Eq. (8). The phonon pulse is present in the time

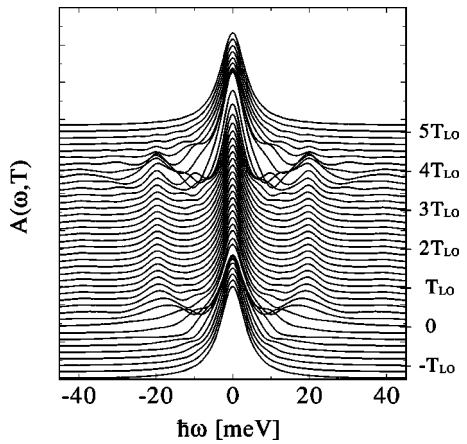


FIG. 2. The time evolution of the nonequilibrium spectral function $A(\omega,T)$ excited by the pulse of the phonon field in the GKB Ansatz, switched in the interval $T=(0,4T_{LO})$. The formation (destruction) of satellites takes roughly a time T_{LO}.

interval $T=(0,4T_{LO})$, where $T_{LO}\approx 200$ fs ($\hbar\omega_0=20$ meV) is the oscillation period of the phonon field. Other parameters are: the level energy $E_0=0$, the coupling $\Gamma=8$ meV, the strength of the electron-phonon coupling $g=0.2$, the nonequilibrium phonon population[23] $n_P(\omega_0)=1$ and the temperature of the reservoirs $T_{\rm latt}=30$ K. Since the change $A(\omega,T)$ depends very little on the values $E_0, T_{\rm latt}$ and the dc bias, we take the equilibrium values $\mu_L=\mu_R=0$. The correlation functions $G^<(\omega,T)$, $G^>(\omega,T)$ are slightly more sensitive to the dc bias, but their time evolution essentially follows $A(\omega,T)$. In $A(\omega,T)$ the formation and destruction of satellite peaks can be clearly observed. The first satellite peaks emerge from the main resonance and settle to the position $\omega=\omega_0$ in about $T\approx T_{LO}$. When the pulse ends the evolution of $A(\omega,T)$ toward the noninteracting $A_0(\omega,T)$ is also oscillatory, with positive maxima at the positions of the steady-state peaks and negative minima in between.

In Fig. 3 we show the time evolution of $A(\omega,T)$ in the individual phonon satellites. The main resonance (A_0), first ($4A_1$) and second ($8A_2$) satellites are presented by the solid, dashed, and dotted lines, respectively. The transfer of the spectral weight to the satellites can be understood as a polaron formation. For all peaks this transient process ends approximately at T_{LO}, since only one-phonon processes are present in W_1. In the exact calculations of a 1D model,[3] the spectral and population dynamics related with the second order satellites takes longer than in the first order satellites. In Fig. 3 the formation process of the polaron is accompanied by fast oscillations, with a time period $T_f\approx T_{LO}/3$, practically independent on g or Γ. The spectral function $A(\omega,T)$ determines the t-causal spectral function $\mathcal{A}(\omega,t)$, with a complex time behavior reflected in the tunneling current (see Figs. 4–6).

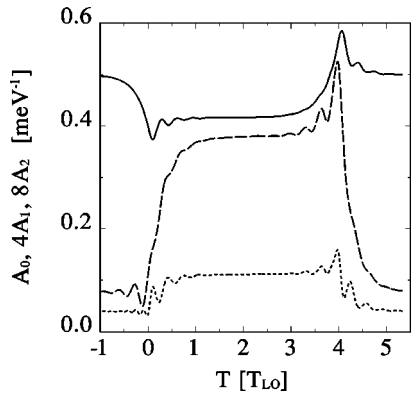


FIG. 3. The time evolution of $A(\omega,T)$ in the individual peaks: the main resonance (A_0), first satellite (A_1, multiplied by 4) and second satellite (A_2, multiplied by 8) are presented by the solid, dashed, and dotted lines. At the pulse edges fast oscillations can be observed.

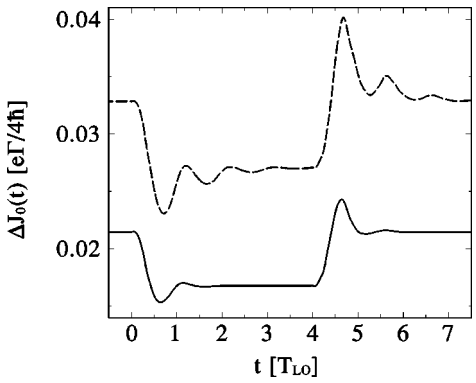


FIG. 4. The tunneling current through the main resonance $\Delta J_0(t)$, calculated for an injection window of the width 4 meV and for the temperature $T=5$ K (30 K), presented by the dashed (solid) line. The fast oscillations wash out with the temperature and the current gets the rectangular form.

B. Phono-induced current

We calculate the induced resonant current $\Delta J(t)$ from (14) for different level positions E_0. These situations are modeled, for simplicity, by the same $A(\omega,T)$, which is just shifted in energy[11] [the transient part of $A(\omega,T)$ depends very little on the level position E_0]. The tunneling window is given by the different chemical potentials $\mu_L=2$ meV, $\mu_R=-2$ meV. In Fig. 4 we show the induced current $\Delta J_0(t)$ through the main resonance $E_0=0$. The dashed (solid) lines correspond to the temperatures $T=5$ K (30 K). As the temperature rises, the oscillations, with the approximate period T_{LO}, become washed out. They are quite reminiscent to

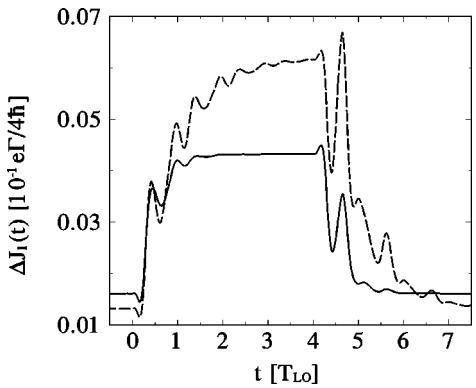


FIG. 5. The tunneling current through the first satellite $\Delta J_1(t)$, calculated as in Fig. 4. The oscillations are twice faster.

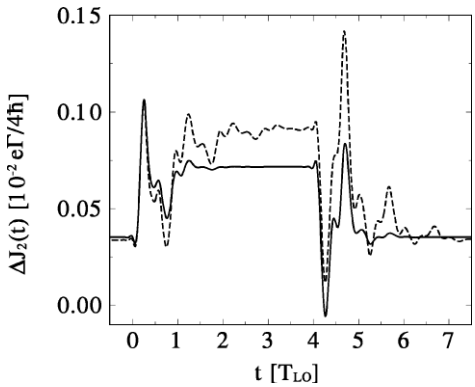


FIG. 6. The tunneling current through the second satellite $\Delta J_2(t)$, calculated as in Fig. 4. Very irregular evolution with beating can be observed.

the ''ringing'' observed in a similar (noninteracting) system in a pulsed electric field.[13] We can check the causality of the current close to the pulse edges $t=0, 4\,T_{LO}$. The oscillations in $J(t)$ survive for long observation times t, in contrast to fast decay of $A(\omega,T)$ as a function of T. The reason is simple: the distribution $\mathcal{A}(\tau,t)$ for large t is connected with $A(\tau,T=t\pm\tau/2)$ for any value of T (it picks up oscillations from early T). The oscillations diminish because of the randomization coming from the width of the injection window, given by the difference $n_{FD}(\hbar\omega-\mu_L)-n_{FD}(\hbar\omega-\mu_R)$, similarly as in Ref. 13. In reality this process could be shorter, since the phonon distribution for this interacting system evolves self-consistently and thus brings some additional randomization. After the pulse ends, damped oscillatory evolution with a similar relaxation time brings the current back to its original value.

In Fig. 5 the induced current through the first satellite $\Delta J_1(t)$ at $E_0=-20$ meV is shown for the same excitation conditions. At $T=5$ K the current saturates relatively slowly with many oscillations with an approximate period of $T_{LO}/2$. They again disappear at higher temperatures, where the response closely follows the rectangular pulse form. Finally, in Fig. 6 we present the induced current through the second satellite $\Delta J_2(t)$ at $E_0=-40$ meV. Its time evolution is more complex, with beatings of the approximate periods $T_{LO}, T_{LO}/2$. The oscillations are very large at both edges of the pulse, where they can shortly give negative values to the transient current. These negative current overshoots are especially large if the dc bias is tuned between the satellites, where they balance the positive transient values at the main resonance and satellites. Because the sum rules are fulfilled at any time, the total current through the resonance for a very wide window is independent on time.

V. CONCLUSION

We have studied resonant tunneling through a single level in the presence of pulsed LO phonon field. The analysis has been conducted by the nonequilibrium linked cluster expansion,[12] generalized to the time domain. The distribution function for the pulsed phonon field, with a zero coherent component, has been constructed and the phonon correlation functions were described by the GKB Ansatz. A detailed examination shows that this Ansatz is necessary in order to obtain an explicitly causal formula for the time-dependent tunneling current.

Numerical results for the nonequilibrium electron spectral function have been obtained by the NLCET method in the first order cluster approximation. They reveal the formation and collapse of satellite peaks on an ultrashort time scale. The transient current through the individual peaks, related with this polaron formation processes, has been calculated. At low temperatures of the injected electrons, it shows oscillations with the phonon frequency and its harmonics. At high temperatures these oscillations become washed out, so that the current follows the rectangular shape of the phonon pulse. Our results indicate that considerable insight about the microscopic details of electron-phonon interactions can be obtained in the time domain. We hope that they can stimulate future experiments with short phonon pulses.

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APPENDIX

Here we use the nonequilibrium distribution $f_P(\omega_0,T)$ in (7) to construct the nonequilibrium phonon correlation functions $\Delta D^<(\tau,T)$, $\Delta D^>(\tau,T)$. The most straightforward approach results by replacing the equilibrium occupation factors $n_{BE}(\omega_0)$ in (6) by $f_P(\omega_0,T)$ and assuming that T is the *average* time between t_1 and t_2 $[T=(t_1+t_2)/2]$. Equivalent, but more compact description, uses in (6) both positive and negative frequencies, and defines $f_P(-\omega_0,T)=n_{BE}(-\omega_0)+\Delta_{-\omega_0}(T)$, where $\Delta_{-\omega_0}(T)=-\Delta_{\omega_0}(T)$. We denote the parts of the free phonon spectral function[17] related with positive/negative frequencies by $A_{D\pm}(\omega)=\pm 2\pi\delta(\omega\mp\omega_0)/\hbar$ and their Fourier transformed counterparts by $A_{D\pm}(\tau)=\pm(1/\hbar)e^{\mp i\omega_0\tau}$. The substitution of $f_P(\pm\omega_0,T)$ in (6) then gives the KB Ansatz[15] for the transient change of the phonon correlation functions and propagators

$$\Delta D^<(\tau,T)=\Delta D^>(\tau,T)$$
$$\approx A_{D\pm}(\tau)\Delta_{\pm\omega_0}(T)=\frac{2}{\hbar}\cos(\omega_0\tau)\Delta_{\omega_0}(T),$$
$$\Delta D^{r,a}(\tau,T)=\mp i\,\theta(\pm\tau)[\Delta D^>(\tau,T)-\Delta D^<(\tau,T)]=0. \tag{A1}$$

Here the product $A_{D\pm}(\tau)\Delta_{\pm\omega_0}(T)$ is

$$A_{D\pm}(\tau)\Delta_{\pm\omega_0}(T)=A_{D+}(\tau)\Delta_{\omega_0}(T)+A_{D-}(\tau)\Delta_{-\omega_0}(T)$$
$$=\frac{2}{\hbar}\cos(\omega_0\tau)\Delta_{\omega_0}(T), \tag{A2}$$

where the definition of $A_{D\pm}(\tau)$ and $\Delta_{-\omega_0}(T)$ are used. The phonon functions in (A1) are causal in the central variable T, but when applied in the NLCET method, they lead to some unphysical results.

It has been known since the work of Lipavský *et al.*,[22] that the KB Ansatz does not fulfill the requirement of cau-

sality of the correlation functions in the individual times t_1, t_2 [i.e., $\Delta D^{<,>}(\tau,T) \neq 0$ for $T \in (0,T_0)$ even if t_1 or t_2 are *out* of $(0,T_0)$]. The problem results from identification of the time T in $f_P(\omega_0,T)$ with the central time $(t_1+t_2)/2$. The simplest correction is called the GKB Ansatz,[22] and we show in the main text that it can give plausible results also in the NLCET method.

In this Ansatz, the phonon spectrum $A_D(\tau)=i[D^r(t_1-t_2)-D^a(t_1-t_2)]$ from the nonequilibrium correlation functions $\Delta D^<$ and $\Delta D^>$ is resolved into two components, propagating forwards and backwards in time (D^r, D^a), respectively. These are multiplied by the population change $\Delta_{\pm\omega_0}$ with the *initial* time coordinates t_1, t_2, pertinent to each of the two terms, instead of the center of mass $(t_1+t_2)/2$

$$\Delta D^<(t_1,t_2) \approx iD^r_{\pm}(t_1-t_2)\Delta_{\pm\omega_0}(t_2) - i\Delta_{\pm\omega_0}(t_1)\ D^a_{\pm}(t_1-t_2). \tag{A3}$$

Here the spectral function $A_D(\tau)$ in the propagators $D^{r,a}(\tau)=\mp i\,\theta(\pm\tau)A_D(\tau)$ is again resolved in the positive/negative frequency parts $A_{D\pm}(\tau)$ and the related propagator components $D^{r,a}_{\pm}(t_1-t_2)$ are multiplied by the factors $\Delta_{\pm\omega_0}(t_{1,2})$ as in (A2). This gives the functions $\Delta D^{<,>}(\tau,T)$ in the form

$$\Delta D^<(\tau,T)=\Delta D^>(\tau,T) \approx \frac{2}{\hbar}\cos(\omega_0\tau)\left[\theta(\tau)\Delta_{\omega_0}\left(T-\frac{\tau}{2}\right) + \theta(-\tau)\Delta_{\omega_0}\left(T+\frac{\tau}{2}\right)\right]. \tag{A4}$$

They reduce to the KB Ansatz from (A1) if the shifts $\pm\tau/2$ of the step function $\Delta_{\omega_0}(T\pm\tau/2)$ are neglected.

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[23] In the present case, the factor $\Delta F_1^<(\tau,T)$ depends on the product $M^2 n_P(\omega_0)$. Therefore, an increase of the nonequilibrium phonon population $n_P(\omega_0)$ can be also interpreted as an increased coupling $g=(M/\hbar\omega_0)^2$. Of course, the equilibrium solution $G^<_{\rm equil}$, which depends through $F_1^<(\tau)$ only on M^2, is different in the two cases. But in our study, focused on the induced change of spectra, we can assume for simplicity that $G^<_{\rm equil} = G_0^<$.

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